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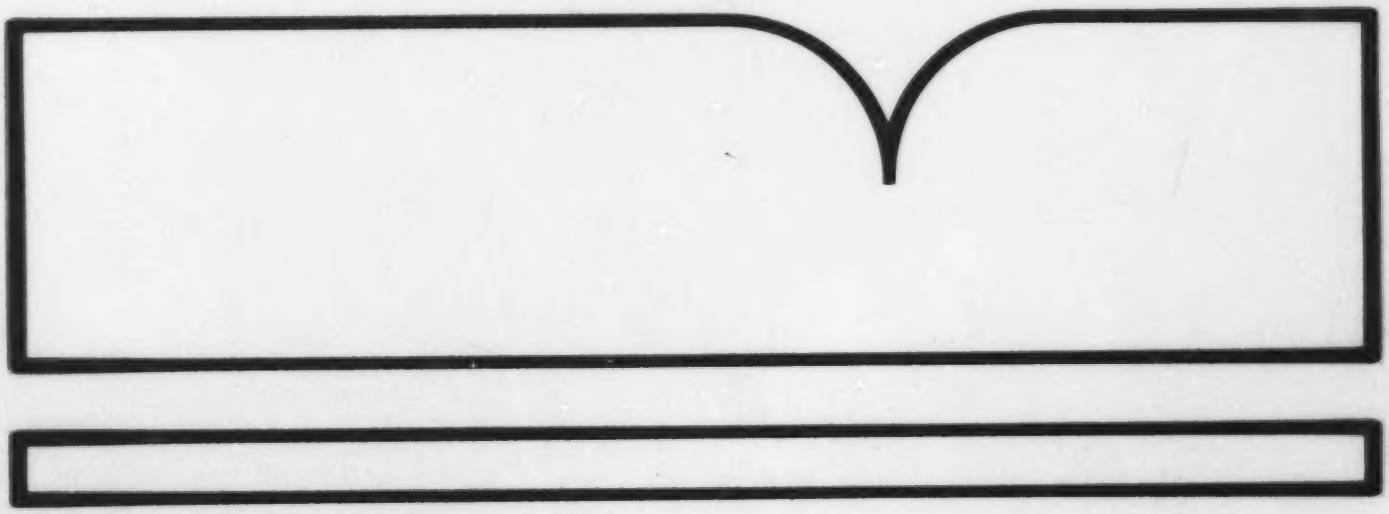

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16. ABSTRACT

This article is a technical commentary relating to the article, "An Advection-Diffusion Concept for Solute Transport in Heterogenous Unconsolidated Geological Deposits," by Gillham, et al, Water Resources Research 20(3):369-378, 1984. The authors principal comments relate to the confusion which has developed in ground water dispersion literature because of the failure to consider properly the two and three dimensional aspects of heterogenous advection and transverse diffusion on the transport of conservative solutes in layered unconsolidated geological deposits.

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JOURNAL ARTICLE

Comment on "An Advection-Diffusion Concept for Solute Transport in Heterogeneous Unconsolidated Geological Deposits" by Gillham et al.

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The paper by *Gillham et al.* [1984] provides valuable insight into some of the combined effects of nonuniform (heterogeneous) advection and transverse diffusion on the transport of conservative solutes in layered unconsolidated geological deposits. We have read this paper with interest since it illustrates, by means of some useful examples (e.g., Figure 2, p. 371), several important features of solute transport in layered media and also because it is closely related to our own recent theoretical work regarding the dispersion of conservative solutes in stratified aquifers [*Molz et al.*, 1983; *Güven et al.*, 1984]. However, we are concerned with certain aspects of the paper, particularly those dealing with a consistent definition for the symbol α_L, the meaning of the phrase "classical advection-dispersion theory," and the apparent deemphasis of the importance of heterogeneous advection on the development of concentration plumes in porous media. At the present time there is an increasing amount of relatively new terminology in the groundwater transport field, and the potential for confusion is high (for a recent example see *Simmons* [1984]). Therefore we believe that a few comments of a clarifying nature will be useful.

As a basis for the following discussion we will use (1) and (2) of *Gillham et al.* [1984] which may be written as

$$\frac{\partial c}{\partial t} + \frac{\partial (v_i c)}{\partial x_i} - \frac{\partial}{\partial x_i}\left(D_{ij}\frac{\partial c}{\partial x_j}\right) = 0 \qquad (1)$$

and

$$D_{ij} = \alpha_T |v| \delta_{ij} + (\alpha_L - \alpha_T) v_i v_j / |v| + D^* \delta_{ij} \qquad (2)$$

where c is solute concentration; $|v|$ is velocity magnitude; v_i $(i = 1, 2, 3)$ is the seepage velocity; D_{ij} is the hydrodynamic dispersion tensor; D^* is the effective molecular diffusion coefficient; δ_{ij} is the identity tensor; α_L and α_T are the longitudinal and transverse principal components of the dispersivity tensor, respectively; t is time; and x_i $(i = 1, 2, 3)$ are Cartesian coordinates. Equations (1) and (2) apply to heterogeneous isotropic porous media with constant porosity; therefore our comments are limited to such conditions.

Most hydrologists would agree that (1) and (2) define the classical advection-dispersion theory. The dimensionality of the equations required to make a realistic application of the theory is determined largely by the nature of the nonuniform flow field $v_i(x_i)$. One can not expect a one-dimensional or two-dimensional form of (1) to describe a flow and transport situation, respectively, that is inherently two dimensional or three dimensional. Given that this is true, it is important to note that virtually all of the calibrated models mentioned by *Gillham et al.* [1984, p. 369] that yielded apparent longitudinal dispersivities in the range of 1–100 m were based on the implicit assumption that the seepage velocity was uniform at least across a vertical section. As we have outlined in the work by *Molz et al.* [1983] and developed more completely in the work by *Güven et al.* [1984], when such an assumption is applied to a stratified aquifer with a nonuniform velocity distribution, the apparent longitudinal dispersivity can grow to extremely large values. In fact, as *Gillham et al.* [1984, p. 370] correctly state the apparent dispersivity is not an invariant property of the medium at all. However, contrary to what is stated by *Gillham et al.* [1984, p. 370] this does not necessarily mean that the advection-dispersion equation, which we understand to be (1), is invalid. It could simply mean that the assumption of a uniform seepage velocity distribution over some part or the whole of the medium is not valid. It is important to note that the dispersivity α_L in (2) is a local mechanical dispersivity that operates at the scale of the repesentative elementary volume used to derive (1). It is not the same as the apparent dispersivity α_L discussed by *Gillham et al.* [1984] in several paragraphs following their (2). In general, this apparent α_L has an ambiguous physical meaning and would depend on the entire three-dimensional flow field as well as on the initial and boundary conditions [*Molz et al.*, 1983; *Güven et al.*, 1984].

At least two recent field experiments have been performed that did not assume a uniform seepage velocity across a vertical section [*Pickens and Grisak*, 1981; *Killey and Moltyaner*, 1984; *Moltyaner and Killey*, 1984]. In both cases the resulting dispersivities were only slightly larger than values measured in the laboratory. Thus field experiments performed properly seem to provide a measure of support for the classical advection-dispersion theory as defined by (1) and (2).

On page 370 of their paper, *Gillham et al.* [1984] state that recent stochastic analyses have shown that the solute spreading process cannot be generally described by classical Fickian transport. This however, can not be viewed as showing that (1) and (2) do not hold when applied in their correct dimensional form. In fact, all of these analyses are based on the assumption that (1) and (2) are valid at the scale of the representative elementary volume.

On page 372, in their discussion of the results obtained for the situation depicted in their Figure 2, *Gillham et al.* [1984] state that "transverse diffusion alone can cause significant longitudinal spreading of a tracer together with a rapid decline in peak concentration values and a significant reduction in the bulk rate of travel of the tracer zone in the permeable layers; effects that are contrary to the advection-dispersion theory." We agree with everything stated except that the effects are necessarily contrary to advection-dispersion theory. Equation (2) includes diffusion as well as mechanical dispersion. If applied in its proper dimensional form (in this case two-dimensional), (1), together with (2), will encompass all of the effects discussed by *Gillham et al.* [1984]. In fact, (3) and (5) used by Gillham et al. to model the situation depicted in Figure 2 are simplified forms of the advection dispersion equation (1). Only if one attempts to fit a one-dimensional form of the advection-dispersion equation to the inherently two-dimensional flow system will problems like those illustrated in

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Figure 3 of Gillham et al. arise [*Molz et al.*, 1983; *Güven et al.*, 1984]. For the α_L in their equation (7) to have the same meaning as the α_L in (2) one must be dealing with a uniform advection in the z direction throughout the entire x-z plane when applying (2) to the situation depicted in Figure 2 of their paper.

It is also not surprising that *Sudicky et al.* [1983] found that they needed a scale-dependent dispersion coefficient to fit their data. The analytical model allowed for dispersion of the injected tracer slug in three dimensions in an infinite medium, but the advective velocity field was assumed to be uniform and one-dimensional while, in fact, it was nonuniform and three dimensional.

Gillham et al. [1984] refer to the combination of the nonuniform advection and transverse diffusion processes as the advection-diffusion process and propose the advection-diffusion model as a physical basis for explaining solute spreading phenomena in layered media. While we mainly agree with them on this point [*Molz et al.*, 1983; *Güven et al.*, 1984], we would like to point out that Gillham et al. seem to emphasize mainly the importance of transverse diffusion in the examples considered in their paper and in their discussion, but they do not seem to give sufficient space to the role of nonuniform advection. We would also like to point out that more detailed results and discussions about the combined effects of nonuniform advection and local transverse mixing (either by diffusion alone or by both diffusion and local mechanical dispersion) are presented in our recent work [*Molz et al.*, 1983; *Güven et al.*, 1984] as well as the recent work of *Sudicky* [1983]. There is also a large body of information on the subject available from the previous studies on mass transport in shear flows in conduits (tubes, pipes, open channels) and in the atmospheric boundary layer (see, for example, *Saffman* [1962], *Monin and Yaglom* [1971], *Fischer et al.* [1979], *Harris* [1979], and *Güven et al.* [1984] for additional specific references). All of these recent and previous studies show that both nonuniform advection and local transverse mixing have an important role in the overall solute transport process in nonuniform advection fields. In fact, strongly heterogeneous advection patterns may lead to "nonsmooth," "non-Gaussian" phenomena like multimodal plumes [*Güven et al.*, 1984] and even complete plume splitting as observed in the tracer test referred to in the paper by *Gillham et al.* [1984, p. 374]. Similarly, very complex plume behaviors, primarily due to complex large-scale heterogeneous advection patterns, may develop in real media as shown in a recent paper by *Osiensky et al.* [1984]. It may also be noted that even in the simple situation considered by *Gillham et al.* [1984, Figure 2], the plume behavior depicted in their Figure 2*c* would be quite different if the permeable layer sandwiched between the two seminfinite impermeable regions were composed of, say, two distinct layers of equal thickness $b = 0.05$ m but with different seepage velocities in each layer, say, 0.05 m/day in one layer and 0.15 m/day in the other, instead of 0.10 m/day throughout the permeable zone as assumed in the example depicted in Figure 2*c*; still another plume behavior would be obtained if the velocities were 0.025 and 0.175 m/day in the respective layers of the permeable zone. In view of this, we would like to emphasize again that reliable modeling of solute transport in real situations would depend crucially on a sufficiently detailed knowledge of not only the local dispersive and solute storage characteristics of the medium, but also a knowledge of at least the major features of the three-dimensional advection field encountered by every part of a solute plume throughout the migration path of the plume. Similar views have been expressed previously by *Smith and Schwartz* [1980, 1981*a*], *Matheron and de Marsily* [1980] and also discussed in our recent work [*Molz et al.*, 1983; *Güven et al.*, 1984].

In summary, it appears to us that some confusion has developed in the groundwater dispersion literature because of failure to consider properly the two- and three-dimensional aspects of heterogeneous advection. In order to test the classical advection-dispersion theory, one must first specify the advection field accurately. In the past when this was done, remarkably small dispersivities resulted. Previous studies, especially in conduits and the atmospheric boundary layer, have shown that when heterogeneous advection fields exist, they usually play a major role in the development of tracer plumes.

Of course, we have said nothing here concerning how one goes about measuring or calculating complex advection fields in aquifers or how one applies the advection-dispersion theory, even if it is valid, in large domains with dimensions of kilometers or tens of kilometers. This is another question entirely that is being studied intently by many investigators at the present time.

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